An Enhanced Discretisation Method for Storage Tank Models within Energy Systems

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Abstract

This article presents a new dicretisation function that can be applied to flow models using Finite-Volume-Method. The function is required since the commonly applied UPWIND disrectisation yields a low accuracy when convection is small with regard to volume size, e. g. for storage tank models. The new approach is compared to measurement data and it shows a much higher accuracy incorporating the same number of control volumes so that the user may decrease the problem size considerably.

Keywords: discretisation method; UPWIND; tank model; cogeneration; energy system

1 Introduction

By means of simulation tools like HKSim [1] it is possible to model complex energy supply system layouts in order to find improvement potential in the development or optimisation phase of a project. HKSim integrates Dymola/Modelica for modelling and simulation. It is used by Imtech Deutschland GmbH & Co. KG and is currently extended by XRG Simulation GmbH for performing simulation of steam and compressed air systems.

Storage tanks are a necessary part of each cogeneration and regenerative power supply system. Those energy systems like shown in Fig. 1 have become very popular for large but also small applications. From the energetic point of view it is of big interest to model the transient behaviour of such tanks in order to determine temperatures in feed and return lines precisely. Since control algorithms for such plants rely on actual temperatures measured at different tank levels a correct temperature prediction influences the result of a simulation for power supply, switch-on times, etc., strongly.

A hot water storage tank fulfils two functions at the same time. First, hot water can be stored when demand and supply is incoherent (e. g., in cogeneration and regenerative applications). Second, a degree of freedom is introduced to the hydraulic layout so that pumps may operate independently for consumers and heat suppliers. For example, the storage tank in Fig. 1 is charged with hot water when the mass flow rate of water through the CHP is greater than that in the return line (boilers are considered to be off) – this happens when heat demand is low and power is required. On the other hand, a discharge is automatically initiated when the CHP is switched off and consumers are still fed. Due to the positive gradient of the specific volume w.r.t. temperature (above 4°C) a very good separation of water with different temperatures can be achieved when the tank is fed with hot medium at top and colder medium at bottom level. For that reason this kind of storage tank always shows an aspect ratio which is greater in vertical direction.

In general, a more or less sophisticated system control (cascade connexion) is implemented which switches CHP and boiler with regard to the actual heat demand and tank temperatures. At low heating demand the tank outlet temperature will rise after a period of time and therefore the CHP has to be shutdown to prevent an overheat of the engine. Of course a switch-on is possible again when heat demand rises but the shutdown interval and the number of power-up sequences influences the durability of the engine (note, that the CHP's are usually adapted from vehicle engines which have an average lifetime of a few thousands of hours). It is therefore of big interest to decrease the number of (cold-)starts at a maximum power output. An optimum performance can be achieved by also defining the "right" tank size. This can be done by means of the simulation tool HKSim.

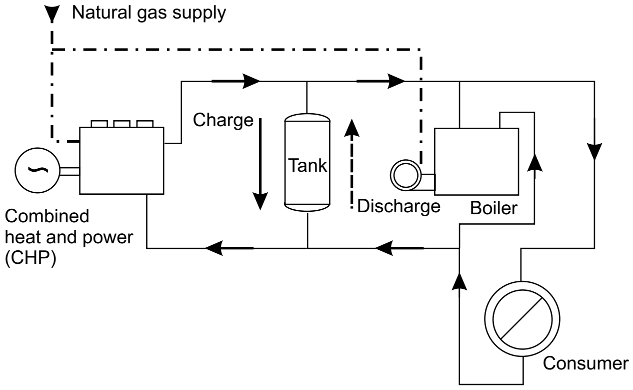

Fig. 1: Simplified schematic of a cogeneration plant with integrated hot water storage tank

2 Discretisation methods for thermo-hydraulic system modelling

Physical systems are always described by partial differential equations which consist of derivatives w.r.t. time and space. Applying simulation tools like Dymola/Modelica it is possible to model and solve the dynamic part of those equations – but all derivatives w.r.t. space have to be simplified using finite methods (e.g., Finite-Volume-Method or Finite-Element-Method [2]).

The balance equations for an incompressible medium with constant density (e.g. liquid water) can be derived from volume integrals [3]. The mass balance is rather simple due to constant density and constant volume size.

$$\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} = 0 \qquad (1)$$

For the energy balance follows assuming low kinetic energy (total energy ≈ internal energy U):

$$\frac{dU}{dt} = \underbrace{\dot{m}_{in} \cdot h_{in} - \dot{m}_{out} \cdot h_{out}}_{\text{convection}} + \underbrace{\dot{Q}_s}_{\text{heat source/sink}} \quad . (2)$$

For incompressible media the specific internal energy $u = U/m$ is a function of temperature as well as the specific enthalpy h (since the density is considered to be constant). Both variables are computed by the specific heat capacity c which is equal to the derivative of specific internal energy u w.r.t. temperature T. The heat capacity may be a function of temperature or can be set constant which is a good approach for pure water in the usual range of operating conditions.

$$U = m \cdot c(T) \cdot T \qquad (3)$$

Therefore one may simplify Eq. (2) to the following term:

$$m \cdot c \cdot \frac{dT}{dt} = \underbrace{\dot{m}_{in} \cdot c \cdot T_{in}}_{\dot{H}_{in}} - \underbrace{\dot{m}_{out} \cdot c \cdot T_{out}}_{\dot{H}_{out}} + \dot{Q}_s \ . (4)$$

Usually, mass and energy balance equations are coupled by a momentum balance but in the incompressible case the pressure is not a parameter for medium properties.

If Eq. (4) has to be solved in order to receive an explicit differential equation one must determine the outflow temperature (or outflow enthalpy) of the volume applying known states calculated for the center of each volume. That is the reason why a discretisation method has to be applied.

Usually, for thermo-hydraulic models like pipes, pumps and heat exchangers an UPWIND discretisation scheme is chosen since it is numerically robust and easy to implement at the same time. Validation reveals that such a discretisation is appropriate for plant components which show a large mixing behaviour [4] (e.g., stirrer tanks). Pipes are modelled by a number of serial control volumes which are connected by the UPWIND discretisation (Fig. 2). The equation for calculating state variables (like specific enthalpy h, temperature T or density d) on **downstream** volume boundaries is simple:

$$\Theta_{down} = \Theta, \quad \Theta = h, T, d \ . (4)$$

But the numerical mixing behaviour of this method shows a large diffusion (refer to temperature slope in 30th and 31st of Aug. in Fig. 3) of heat for tanks which are designed to store hot water in vertical layers. In order to prevent this one has to model each tank with a large number of control volumes (usually more than 40 volumes) if UPWIND is applied. Therefore, the tank model generates a high computational effort when it comes to plant simulations for one year simulation time.

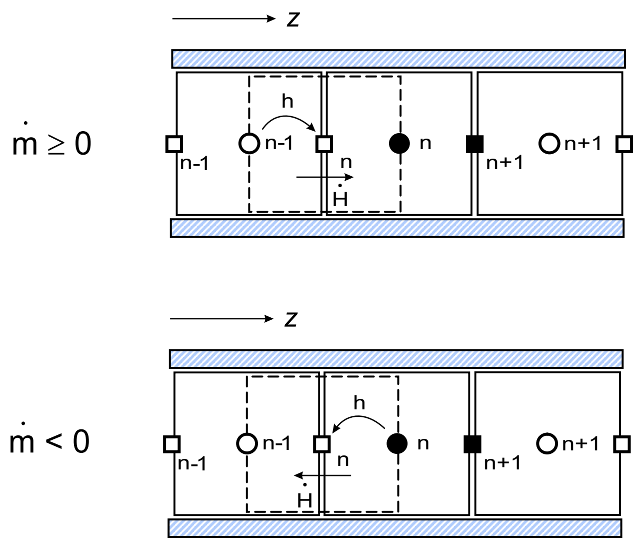

Fig. 2: UPWIND-discretisation for an one-dimensional flow and different directions of mass flow

Another very common discretisation method is the linear interpolation of states between two volumes. Unfortunately, this method results in an unstable or even false solution when it comes to transient simulations. Therefore, UPWIND is still widely used even when storage tanks have to be modelled.

A typical outcome of such a simulation is shown in Fig. 3. One can easily see, that the temperatures during discharge (31st of Aug.) are not predicted very well by the model which consists of 40 discrete volumes. In fact, there is a strong deviation with regard to time. In addition, the bottom temperature is rising quicker than measured during charging periods (28th of Aug.). Since the volume of the tank is known precisely, and the mass flow rate is available from measurements the discretisation method remains as one significant failure potential.

Another indication is following from control theory: Calculating the transfer function of Eq. (4) and furthermore carrying out a Fast Fourier Transformation for a step response the following equation is yielded.

$$T_{out} - T_{out,t=0} = \Delta T_{step,in} \cdot \left(1 - e^{-\frac{t\dot{m}}{m}}\right) \quad (5)$$

The result of such a step response is visualised in Fig. 4. It clearly shows that the state (e. g. temperature) of a finite volume immediately rises due to a sudden change of enthalpy flow at the upstream boundary. Therefore, the outgoing flow changes its temperature **at the same time** the step or any change occurs at the inlet boundary.

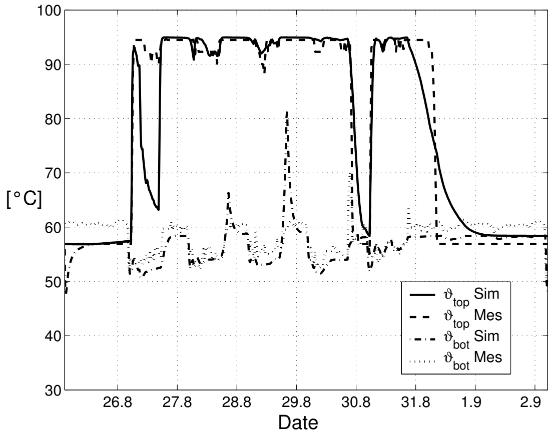

Fig. 3: Comparison of simulation (Sim) and measurement (Mes) of hot water temperatures at bottom and top level of a storage tank – Simulation is carried out by using UPWIND method

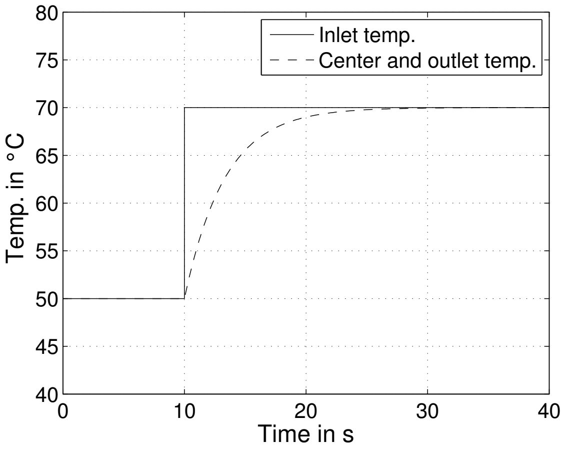

Fig. 4: Step response (Eq. 5) with a weight factor $\dot{m}/m = 0.3$

3 An enhanced discretisation approach

The requirements for a new discretisation method for storage tank models are listed below:

1. Most important, the approach shall deliver a better prediction for enthalpy flows over control volume boundaries w.r.t. time.

2. Standard interfaces which are used for UPWIND models shall be compatible.

The main idea of the new discretisation method is to achieve a gradient-dependent outflow state. Therefore, the gradient between the adjacent upstream control volume and the center state is taken as a criteria for the interpolation between downstream and center state. The interpolation function is chosen to be of exponential type:

$$\Theta_{out} = \left(\Theta_{center} - \Theta_{down}\right) \cdot e^{-a\left|\Theta_{up} - \Theta_{center}\right|} + \Theta_{down} \,. \quad (6)$$

This function is valid for both possible flow directions in one-dimensional flow modelling. While the upstream gradient is not close to zero the outlet boundary state is almost equal to the downstream state. But when the upstream gradient is small the exponential function turns zero and therefore the corresponding volume is considered to be “charged”. From that point the outflow state will be equal to the center state. By increasing the tuning parameter *a* it is possible to decrease the numerical “diffusion” of that method (see Fig. 5).

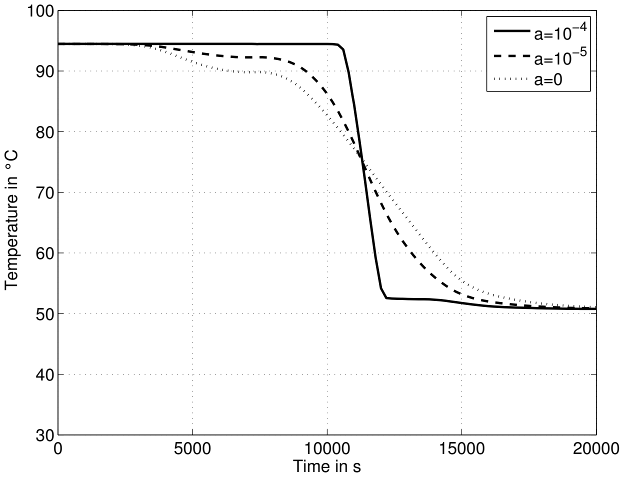

Fig. 5: Discharge temperature of the storage tank model shown in Fig. 6 for different tuning factors *a*

It has to be mentioned that the tank model also displays buoyancy effects (which are an important feature) and therefore the unstable case of different gradient signs w.r.t. upstream and downstream directions is not resulting into problems. If this function should be applied for modelling of other flow problems provisions in Eq. (6) should be made in order to encounter those possible problems.

4 Validation of tank model

For the validation of the tank model measurement data is available. It shows the flow rate through a storage tank as well as the temperatures for top and bottom duct. The tank model looks like displayed in Fig. 6.

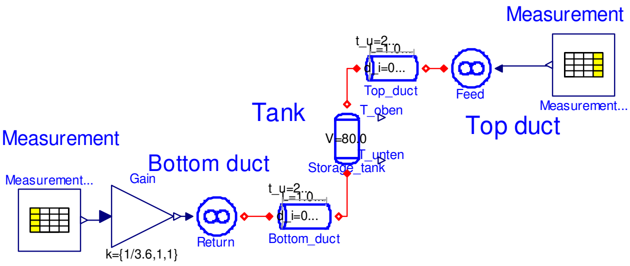

Fig. 6: Validation model for the storage tank model

The storage tank has a liquid volume of 80 m³ and the thermal insulation can be considered to be ideal (adiabatic conditions). Buoyancy effects in flow direction (gravity vector is parallel to flow vector) are taken into account. Since the mass flow rate is provided in terms of a volume flow rate a gain block divides this input by 3.6 (conversion from m³/h to kg/s for water). The measurement data was obtained from the beginning of a heating period in September and was recorded from a large cogeneration plant with a time step interval of 15 min. The tank model applies the new discretisation approach (a=10^{-4}) for $n-4$ volumes. This means that the first two and last two volumes refer to UPWIND-scheme.

An operation of one week is investigated.

Number of volumes *n*	10 new	10 UPW	20 UPW	40 UPW
Non-dimensional simulation time **σ_{sim} [-]**	2.60	1.00	3.92	11.82

Tab. 1: Effect of both discretisation methods and number of volumes on non-dimensional simulation time

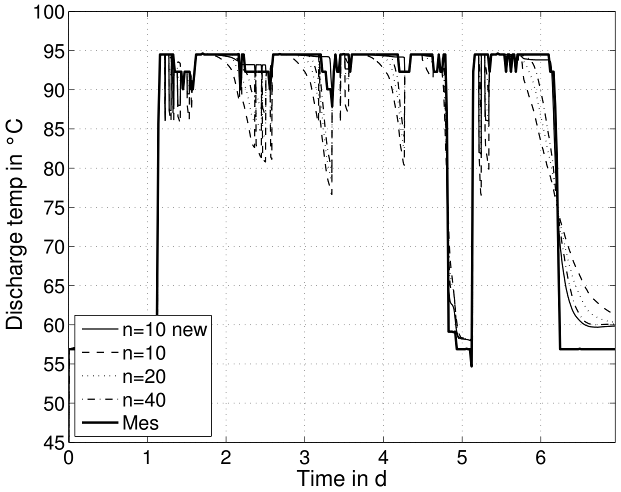

Fig. 7: Comparison of discharge temperature with measurement data

First, the effect of a different number *n* of discrete volumes is evaluated. The result is shown in Fig. 7. One can see that the accuracy of the solution with regard to the measurement of the top duct temperature is rising when the number of volumes is increased. But with the number of control volumes also the simulation time increases as Tab. 1 reveals. Deviations for the top temperatures are high when convection is small w.r.t. volume size (see day 6 to 7). But also the bottom temperature shows higher deviations when the number of control volumes is decreased (refer to peaks in Fig. 8 on 2nd, 3rd and 4th day).

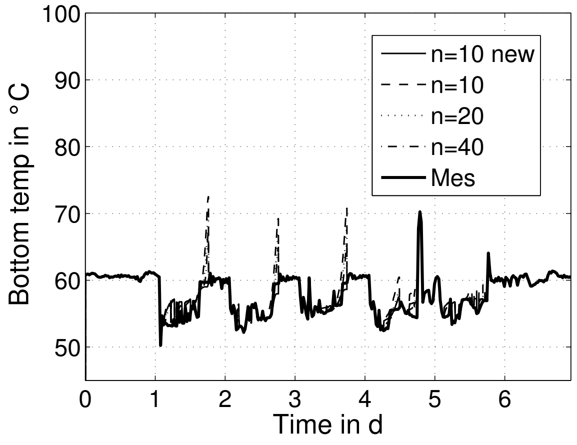

Fig. 8: Comparison of bottom temperature with measurement data

Using the new discretisation it is possible to achieve a much higher accuracy. Comparing Fig. 7 and Fig. 9 reveals that the top temperature is following the measurement much better although just 6 volumes (due to interface compatibility, note remark on previous page) are applying the new discretisation scheme. This fact explains a visible deviation on 7th day in Fig. 9 which is due to the two uppermost volumes using UPWIND discretisation. Adapting hydraulic interfaces in order to enable a calculation of the upstream gradient will lead to a better anticipation of the measurement.

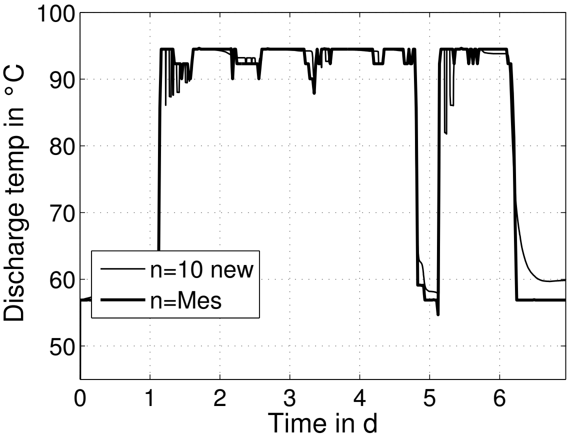

Fig. 9: Comparison of new discretisation approach with measurement data

In addition, also the bottom temperature is predicted more precisely so that the temperature peaks obtained with a discretisation of 10 or 20 volumes vanish (compare Fig. 8 and Fig. 10).

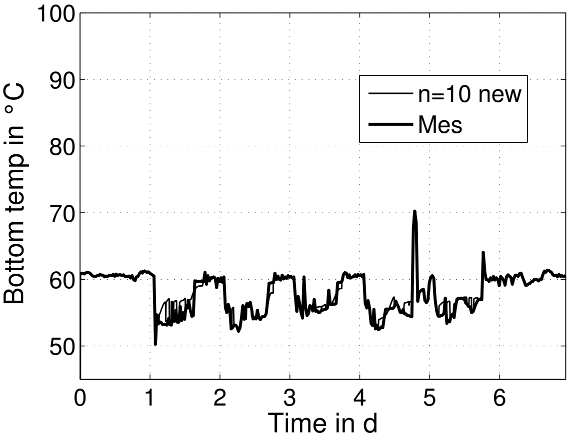

Fig. 10: Comparison of new discretisation approach with measurement data

5 Evaluation for cogeneration plant

What are possible consequences of the discretisation's implementation within a transient simulation model of a typical cogeneration plant? As mentioned before the purpose of a hot water storage tank is to decouple power and heat supply. In case of a high power requirement the tank will be charged until the maximum capacity is reached. At this point CHP's heat production must be cut down or even completely shut down. Thus, possible power peaks can not be decreased resulting in high power charges (actually, it is not the energy rate which dominates in such a case). So, from the energetic point of view one could tend to integrate larger tanks in such plants. But from the financial point of view this strategy is obviously restricted to defined limits. Of course, it is possible to integrate heat exchangers for removing "waste" heat to ambience but this option is not desirable from the energetic point of view.

In Fig. 11 a simulation model of a typical industrial cogeneration plant is shown. Two CHP's are installed for providing power (power priority control) while the boilers are used in order to control the feed temperature and backup heat production in case CHPs are off. The heat consumers are represented by a simple consumer model on the right side of that figure. The profile of the heat consumption may change dynamically and almost any signal source from the Modelica.Blocks library may be used.

In this evaluation a simple constant power and heat demand was applied while the power demand (1200 kW_{el}) is larger than the heat demand (600 kW_{th}) forcing the tank (volume V = 30 m³) to be charged when CHPs are on. A very ordinary flip-flop control is used to prevent overheating at CHP's hot water inlet. Basically, both engines are switched off when a temperature of 87 °C at the bottom level **and** 100 °C at the top is exceeded. A restart is possible again when top temperature drops below 90 °C. The tuning parameter a_{dis} was set to either 0 for UPWIND or 1e-4.

Results reveal that with the same tank volume but different discretisation schemes the operational time at the same power output is different. Actually, the CHP models can be operated 15 to 20 % longer than with UPWIND discretisation (see Fig. 12 and Fig. 13). The difference is depending on the supplied CHP control logic. So, the chosen discretisation may result in considerably smaller tank sizes when a certain operation time interval has to be guaranteed – another evaluation reveals that the same mean continuous operation interval (time between switch on and switch off) is reached with new discretisation when tanks are approx. 25 % smaller in volume. Also, it is important knowing the number of power-up sequences during a certain period of time which has to be lower than the requirement from the CHP manufacturer when warranty conditions shall be respected.

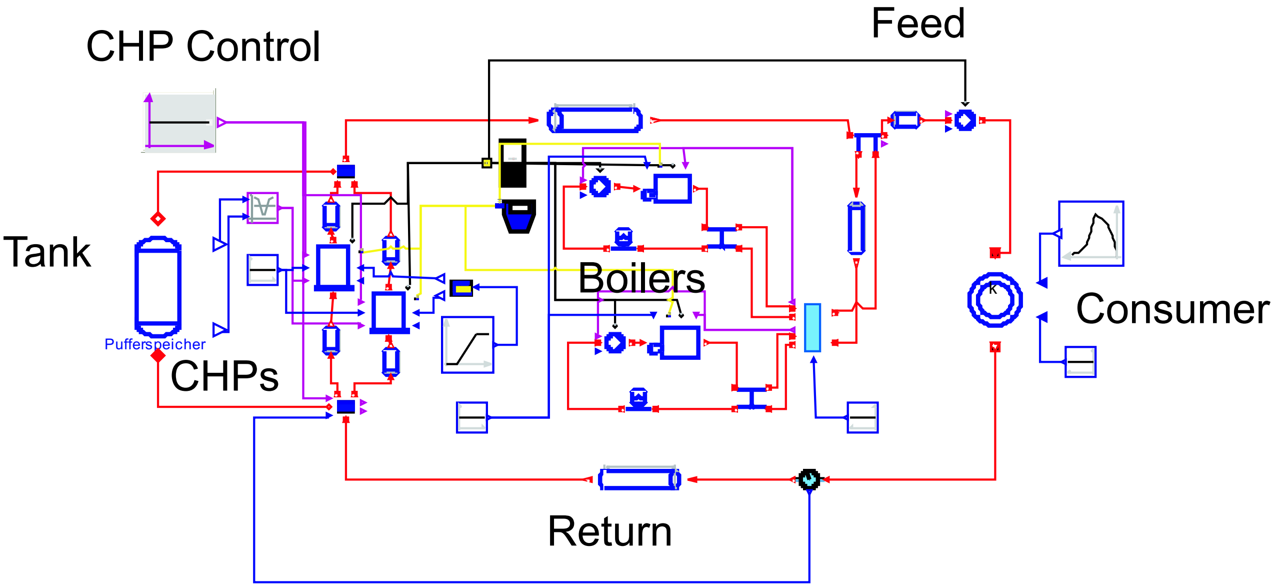

Fig. 11: Simulation model of a cogeneration plant with temperature dependent CHP control

For the simulation of one week the total number of engine starts (for a single engine) was determined with 37 for UPWIND and 31 for new discretisation approach. For both simulations it must be pointed out that the total power and heat production of all moduls was basically equal indicating that the energy balance of the system was conservative (this statement applies also for a simulation of one year under the same boundary conditions).

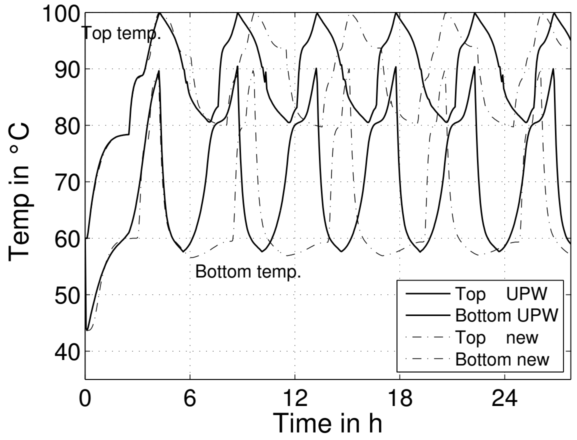

Fig. 12: Tank temperatures for both discretisation approaches

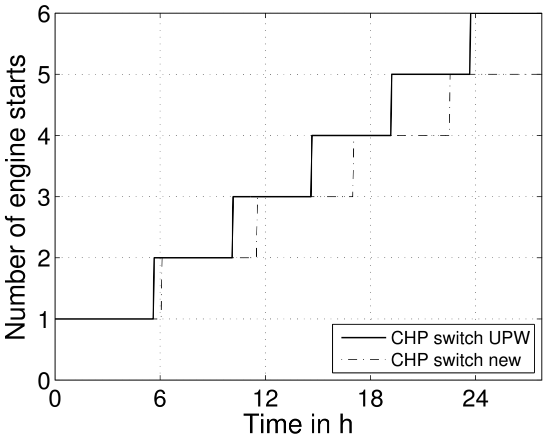

Fig. 13: Number of CHP engine starts

6 Conclusions

The new discretisation scheme enables a more accurate modelling of storage tank models although far less control volumes are required than for a conventional UPWIND discretisation that yields almost same results. Thus, computation times are reduced considerably. Validation shows that temperatures at standard positions for temperature measurements are predicted with a very good agreement. Especially, temperatures during slow discharge are displaying a much sharper gradient than with UPWIND method. It is possible that the new method may also be used for other flow problems which show the same behaviour: convective flow and very low dissipation. (sharp border front flow) like for example in some kind of evaporators in cooling plants. The influence of dissipation which is displayed in the gradient of a step change is adjusted by a single parameter. This tuning parameter can be changed so that mathematically the UPWIND method is applied. It must also be stressed that the method is compatible with control volume models which apply UPWIND. If this method should be applied for every component model the interfaces must be changed in order to access downstream and upstream states for first and last control volume in any component model.

With the new discretisation method it is possible to obtain smaller tank sizes while assuming identical boundary conditions. The result's difference could reduce investment costs when large plants are planned by means of simulation tools. For example the savings in acquiring a 30 m³ instead of a 40 m³ tank could be approx. 8.000 € [5].

References

[1] Lüdemann, B.; Wischhusen, S.; Engel, O.; Schmitz, G.: Optimierte Energiesysteme, BWK, Bd. 55, No. 9, Springer VDI-Verlag, Düsseldorf, Germany, 2003.

[2] Casella F. and Schiavo F.: Modelling and Simulation of Heat Exchangers in Modelica with Finite Element Methods. In Proceedings of 3rd Modelica conference, Linköping, Sweden, pp. 343-352.

[3] Wischhusen, S.: Dynamische Simulation zur wirtschaftlichen Bewertung komplexer Energiesysteme. Cuvillier Verlag, Göttingen, Germany: PhD thesis, Department of Thermodynamics, Hamburg University of Technology, 2005.

[4] Mühlthaler, G. Anwendung objektorientierter Simulationssprachen zur Modellierung von Kraftwerkskomponenten, VDI Verlag, Düsseldorf, Germany: PhD thesis, Department of Thermodynamics, Hamburg University of Technology, 2001.

[5] KFServer – Online-Server for cost functions of energy supply system components: http://kfserver.kaiserstadt.de.